IV. Work and energy -
basic terminology

Work

Work (mechanical work) W is a scalar physical quantity that expresses the degree of trajectory effect of the applied force..

The work is performed by:

- force when it moves a body (mass point) along a trajectory (from a place with a position vector $\vec{r_1}$ to a place with a position vector $\vec{r_2}$),
- body (material point), which acts on another body or. material point (since force is a measure of the interaction of material objects).

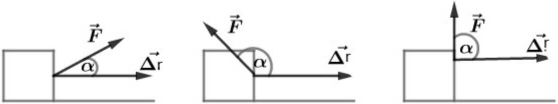
Elementary work ΔW sily $\vec{F}$ is defined as the scalar product of the acting force $\vec{F}$ and the elementary displacement $\Delta\vec{r}$ $(\Delta\vec{r} = |\vec{r}| = \vec{r_2} - \vec{r_1})$.

$$\Delta W = \vec{F} \cdot \Delta\vec{r} = F \cdot \Delta r \cdot \cos\alpha$$

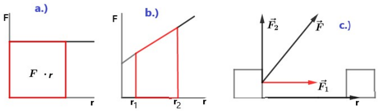
$$[W] = [F] \cdot [r] = 1N \cdot 1m = kg \cdot m^2 \cdot s^{-2} = 1J$$

Joule is the work done by the 1 Newton force acting in the direction of the 1 meter path.

$$1eV = 1,602 \cdot 10^{-19} J$$
$$1kWh = 3,6 \cdot 10^{6} J$$



- if $\alpha < \frac{\pi}{2}$, *the force does the work*
- if $\alpha > \frac{\pi}{2}$, *the force consumes work*
- if $\alpha = \frac{\pi}{2}$, *the force is zero*



a.) If the trajectory is part of a line and the force is constant (it has the direction of the displacement vector), then it valids:

$$W = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\alpha = F \cdot r \cdot \cos 0 = F \cdot r$$

b.) If the trajectory is part of a line and the magnitude of the force changes, then the work is determined by the content of the shape below the force graph (a certain integral) and it valids:

$$W = \int_{r_1}^{r_2} F \,\mathrm{d}r$$

c.) If the trajectory is part of a line and the force is constant (with the displacement vector angle α), then the work is done only by the force component in the direction of the displacement vector:

$$W = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\alpha$$

$$W = \int_{r_1}^{r_2} \vec{F} \cdot \mathrm{d}\vec{r}$$

$$W = \int_{r_1}^{r_2} F \,\mathrm{d}r \cos\alpha$$

$$W = \int_{r_1}^{r_2} F \cos\alpha \,\mathrm{d}r$$

$$W = F\cos\alpha \int_{r_1}^{r_2} \mathrm{d}r = F\cos\alpha\,[r_2 - r_1] = F\cos\alpha\, r$$

$$W = Fr\cos\alpha$$

Energy

Energy E is a scalar state (characterizes the state of an object or event) physical quantity, the change of which ΔE can be determined by the work W performed by the system or the environment. It expresses the extent to which an object (mass point) can exert a force on another object (mass point) and move it along a certain path. So it valids:

$$\Delta E = E_2 - E_1 = W.$$

The energy of the system:

- increases if the work is done by the environment,
- decreases if the work is done by the system.

$$[E] = [W] = N \cdot m = kg \cdot m^2 \cdot s^{-2} = 1J$$

Mechanical energy E changes during mechanical events and characterizes the mechanical state of the system (position and velocity of all particles of the system):

- from the velocity is called kinetic energy E_k,
- from the position is called potential energy E_p.

Kinetic energy of mass point

Consider a mass point with mass m, which is at rest with respect to the inertial frame of reference. If the force $\vec{F}$, begins to act on the mass point, then the mass point will perform (work) a uniformly accelerated rectilinear motion.

$$W = \vec{F} \cdot \vec{r} = Fr\cos\alpha = Fr = (ma)\left(\frac{1}{2}at^2\right) = \frac{1}{2}m(at)^2 = \frac{1}{2}mv^2$$

The work expresses the change in the kinetic energy of a mass point by the action of force $\vec{F}$:

$$\Delta E_k = \frac{1}{2}mv^2$$

$$W = \int_{r_1}^{r_2} \vec{F} \cdot \mathrm{d}\vec{r}$$

$$W = \int_{r_1}^{r_2} m\vec{a} \cdot \mathrm{d}\vec{r}$$

$$W = \int_{r_1}^{r_2} m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \cdot \mathrm{d}\vec{r}$$

$$W = \int_{v_1}^{v_2} m\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \cdot \mathrm{d}\vec{v}$$

$$W = \int_{v_1}^{v_2} m\vec{v} \cdot \mathrm{d}\vec{v}$$

$$W = \int_{v_1}^{v_2} mv\,\mathrm{d}v = m\left[\frac{v^2}{2}\right]_{v_1}^{v_2} = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2 = E_{k2} - E_{k1} = \Delta E_k$$

Gravity potential energy

Consider a mass point with mass m on the Earth's surface. If a mass point moves to h, its position relative to Earth changes. When relocating, work will be done:

$$W = \vec{F} \cdot \vec{r} = (m\vec{g}) \cdot \vec{r} \text{ Nech } r = h.$$

$$W = mgh$$

The work W expresses the change in the potential energy of a mass point:

$$\Delta E_p = mgh$$

$$W = \int_{r_1}^{r_2} \vec{F} \cdot \mathrm{d}\vec{r}$$

$$W = \int_{r_1}^{r_2} (m\vec{g}) \cdot \mathrm{d}\vec{r}$$

$$r = h$$

$$W = \int_{h_1}^{h_2} (m\vec{g}\,) \cdot \mathrm{d}\vec{h}$$

$$W = \int_{h_1}^{h_2} mg\,\mathrm{d}h = mg\,[h]_{h_1}^{h_2} = mgh_2 - mgh_1 = E_{p2} - E_{p1} = \Delta E_p$$

Potential energy of elasticity

When the spring is extended, a force of elasticity arises, the magnitude of which is directly proportional to the elongation l (for a certain elongation), k is the stiffness of the spring and applies:

$$\vec{F_{pruz}} = k\vec{l}$$

Extending the spring changes the relative positions of the spring particles, therefore its energy also changes, which is related to the change in the position of the particles and is called potential energy of elasticity.

If we want to extend the spring by Δl, an external force $\vec{F} = -\vec{F_p}$, which performs the work, must act on the spring $W = \frac{1}{2}k(\Delta l)^2$.

This work expresses the change in the potential energy of elasticity and it valids:

$$\Delta E_{pruz} = \frac{1}{2}k(\Delta l)^2$$

Energy conservation law

The energy of the system is the sum of all forms of energy of all material objects that belong to the system. It changes as the system interacts with the environment.

In events in an isolated system, one form of energy changes into another, but the total energy is constant. This is expressed by the law of conservation of energy.

Energy conservation law:

The total energy of the isolated system (the work of external forces is equal to zero) is constant (does not change with time).

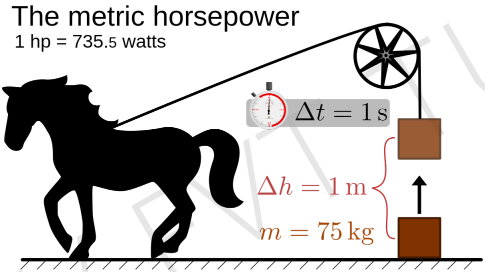
$$E_k + E_p = kont.$$

Performance

performance: $1\,000 kW$ (1 360 koní), mass: $1\,360 kg$

speed: more than $440 km/h$, average: $100 km/h$ for $2,8$ s

price: $1\,032\,000 EUR$



horsepower (hp)

$1\,hp = 735,5\,W$ metrický kôň

Performance P is a scalar quantity that characterizes the speed of work per unit time.

Average performance

$$P_p = \frac{\Delta W}{\Delta t}$$

Immediate performance

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$[P] = \frac{[\Delta W]}{[\Delta t]} = \frac{1J}{1s} = \frac{kg \cdot m^2 \cdot s^{-2}}{1s} = 1W$$

Watt is the performance, at which 1 joule work is performed in 1 second.

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v} = Fv\cos\alpha$$

If we supply energy E to the system for the period Δt and the system does the work W, then

$$P_0 = \frac{E}{\Delta t}$$

is called power consumption.

Efficiency of the system is the ratio of performance and power consumption and it valids:

$$\eta = \frac{P}{P_0} = \frac{\frac{W}{\Delta t}}{\frac{E}{\Delta t}}$$

Efficiency is often expressed in the form of:

$$\eta = \frac{P}{P_0} 100\%$$