I. Kinematics of a mass point - basic terminology

Mechanics is a part of physics that deals with the study of the mechanical motion of material objects.

Kinematics - deals with the motion.

Dynamics - deals with forces.

We divide mechanics in physics according to the object of research into:

a.) mechanics of mass point

b.) mechanics of the system of mass points

c.) mechanics of solid state

b.) mechanics of continuum

Motion and position of a mass point

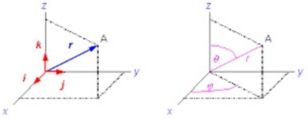
Kinematics (from Greek kineo) - a part of mechanics that examines the state of motion of physical objects, does not deal with the causes of its formation or destruction.

Mechanical motion is a change in the position of a body relative to another body. Motion takes place in space and time.

Mass point is a model of a body in which we do not consider its dimensions and keep only its mass.

The body with which we study this motion is called reference body.

Reference point (beginning of the coordinate system) together with the coordinate system and the timing system form reference system (most often a rectangular coordinate system).

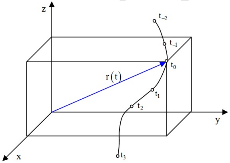


Motion and position of a mass point are described by formulas:

$$x = x(t), y = y(t), z = z(t)$$

$$\vec{r(t)} = x(t) \cdot \vec{i} + y(t) \cdot \vec{j} + z(t) \cdot \vec{k} \ ,$$

kde $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors oriented in the positive direction of the coordinate axes x, y, z.



Trajectory - (from Latin trajectio - transport) is the set of all points to which a mass point reaches during its motion. It is a curve that a mass point describes during its motion.

Distance that a mass point moves over a period of time is the length of the trajectory.

Velocity and acceleration

Average velocity v_p expresses the ratio of the distance (Δs) between points A and B and time (Δt), for which the mass point passes from place A to place B.

Average velocity

$$v_p = \frac{\Delta s}{\Delta t}$$

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{d\left[x(t)\cdot\vec{i} + y(t)\cdot\vec{j} + z(t)\cdot\vec{k}\right]}{dt}$$

$$\vec{v} = \frac{dx}{dt}\cdot\vec{i} + \frac{dy}{dt}\cdot\vec{j} + \frac{dz}{dt}\cdot\vec{k}$$

Simplified formulas for calculating examples:

$$v(t) = \frac{ds}{dt}$$

$$s(t) = \int v(t)\,\mathrm{d}t$$

$$[v] = \frac{[\Delta s]}{[\Delta t]} = \frac{1m}{1s} = 1m \cdot s^{-1}$$

$$1\frac{km}{h} = \frac{1000m}{3600s} = 0,27\frac{m}{s} = 0,27m \cdot s^{-1}$$

$$1\frac{m}{s} = \frac{\frac{1}{1000}km}{\frac{1}{3600}h} = 3,6\frac{km}{h} = 3,6km \cdot h^{-1}$$

Accelaration $\vec{a}$ characterizes the change of movement state (change of velocity per unit of time).

If the mass point had in time t velocity v_1 and in the time $t + \Delta t$ velocity v_2, then average accelaration $\vec{a_p}$ is:

Average accelaration

$$\vec{a_p} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous accelaration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Simplified formulas for calculating examples:

$$a(t) = \frac{dv}{dt}$$

$$v(t) = \int a(t)\,\mathrm{d}t$$

$$[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{1\frac{m}{s}}{1s} = 1\frac{m}{s^2} = 1m \cdot s^{-2}$$

Types of motions

Uniform rectilinear motion

Uniform rectilinear motion is a motion in which the velocity is constant, the magnitude and direction of the velocity do not change over time.

$$s(t) = \int v(t)\,\mathrm{d}t = v\int 1\,\mathrm{d}t = vt + c; c \in R$$

In the time $t = 0 \Rightarrow$

$$s(0) = v \cdot 0 + c \Rightarrow c = s(0) = s_0 \Rightarrow s(t) = s_0 + vt$$

Non - uniformly rectilinear motion

Non - uniformly rectilinear motion is a motion in which the magnitude of the acceleration is constant but non-zero. Size or direction of instantaneous acceleration does not change over time.

If the vectors $\vec{v}$ and $\vec{a}$ have the same direction: $v(t) = v_0 + at$. We call such a motion uniformly accelerated motion.

$$s(t) = \int v(t)\,\mathrm{d}t = \int (v_0 + at)\,\mathrm{d}t = v_0 t + \frac{at^2}{2} + c \Rightarrow (c = s_0)$$

$$s(t) = s_0 + v_0 t + \frac{at^2}{2}$$

If the vectors $\vec{v}$ a $\vec{a}$ have not the same direction: $v(t) = v_0 - at$. We call such a movement uniformly slowed motion.

$$s(t) = s_0 + v_0 t - \frac{at^2}{2}$$

Free fall

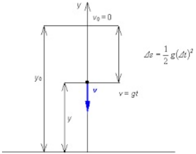
Free fall is a motion in which a body is released freely from a height to Earth in a vacuum. Free fall from a height that is small compared to the dimensions of the Earth, the movement is evenly accelerated.

Acceleration of free fall is called gravity acceleration $g = 9,80665 m.s^{-2}$.

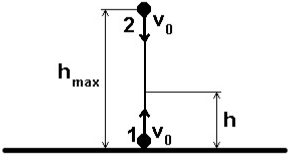
It is uniformly accelerated motion ($a = g$) with zero initial velocityd ($v_0 = 0$) and ($s_0 = 0$), so valid:

$$v = gt$$

$$s = \frac{1}{2}gt^2$$



Vertical throw down

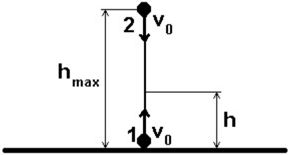


Vertical throw down is a uniformly accelerated rectilinear motion ($a = g$), in which a body, at a height of h above the ground, is thrown vertically downwards at the initial velocity v_0.

$$v = v_0 + gt$$

$$s = v_0 t + \frac{1}{2} g t^2$$

Vertical throw up

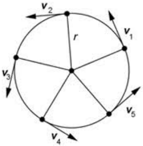


Vertical throw up is a uniformly slowed rectilinear movement ($a = g$), in which the body, at the surface of the earth, is thrown vertically upwards to a height h at the initial velocity v_0.

$$v = v_0 - gt$$

$$s = v_0 t - \frac{1}{2} g t^2$$

Uniformly motion along a circle



Uniformly motion along a circle or rotational motion is a motion in which a mass point (body) moves in a circle with a constant magnitude of velocity (velocity direction is not constant), $v = konst.$, $\vec{v} \neq konst.$

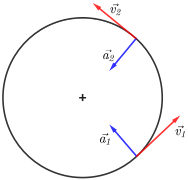
It is a periodic motion that is characterized by period T and frequency f.

Period T is the time for which mass point performs one circle cycle.

Frequency f is the inverse of the period. Its unit is hertz (Hz).

$$f = \frac{1}{T}$$

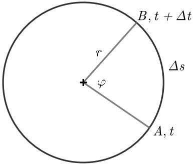
$$[f] = \frac{1}{[T]} = \frac{1}{1s} = 1s^{-1} = 1Hz$$



The instantaneous circumferential velocity $\vec{v}$ has the direction of the tangent to the circle at each point, so the direction of the velocity changes.

Acceleration with uniform motion along a circle is called centripetal acceleration a_d, it is valid:

$$a_d = \frac{v^2}{r}$$



With a uniform motion along a circle, a mass point over time Δt goes direction Δs. For the size of its circumferential speed is valid:

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} = 2\pi r f.$$

If the mass point at the moving along a circle (with radius r) for a time Δt goes direction Δs, then its guide will describe plane angle φ with magnitude:

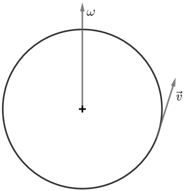
$$\Delta\varphi = \frac{\Delta s}{r}.$$

Rotation vector $\vec{\varphi}$ is a vector that unambiguously expresses the position and orientation of the plane angle φ in space.

- It is perpendicular to the plane of motion of a mass point.
- It is the same size as the plane angle φ.
- It is oriented in the direction from which the rotation of the position vector of the mass point appears counterclockwise.

Instantaneous angular velocity $\vec{\omega}$ of mass point in the time t is vector quantity and it is valid:

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\varphi}}{\Delta t} = \frac{d\vec{\varphi}}{dt}.$$



When the instantaneous angular velocity of a mass point changes during its motion, the mass point has a non-zero angular acceleration.

Instantaneous angular acceleration $\vec{\epsilon}$ in the time t is:

$$\vec{\epsilon} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt} = \frac{d}{dt}\left(\frac{d\vec{\varphi}}{dt}\right) = \frac{d^2\vec{\varphi}}{dt^2}$$

Simplified formulas for calculating examples:

$$f = \frac{1}{T} \Rightarrow T = \frac{1}{f}$$

$$a_d = \frac{v^2}{r}$$

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{\frac{\Delta s}{r}}{\Delta t} = \frac{1}{r} \cdot \frac{\Delta s}{\Delta t} = \frac{v}{r}$$

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

$$v = \omega \cdot r,\ v = 2\pi f r,\ v = \frac{2\pi r}{T}$$

$$[\varphi] = rad$$

$$[v] = m \cdot s^{-1}$$

$$[\omega] = \frac{[d\varphi]}{[dt]} = \frac{1rad}{1s} = 1rad.s^{-1} \text{ (or } s^{-1}\text{)}$$

$$[a_d] = m \cdot s^{-2}$$

$$[\epsilon] = \frac{[d\omega]}{[dt]} = \frac{\left[d^2\varphi\right]}{[dt^2]} = \frac{1rad}{1s^2} = 1rad.s^{-2}$$