Cart on a Ramp

Physical principle.

Kinematic of a material point

Material point – a model of a body in which we neglect its dimensions and maintain its weight.

Trajectory – the sum of all positions that a material point gradually follows during its movement.

Average velocity ($\mathcal{V}_s$) – a vector quantity defined as dividing the path (distance) S by the time t it took a material point to pass this distance.

$$\mathcal{V}_s = \frac{\Delta S}{\Delta t}$$

$$[\mathcal{V}_s] = \frac{[\Delta S]}{[\Delta t]} = \frac{1m}{1s} = 1m * s^{-1}$$

Instantaneous velocity ($\mathcal{V}_o$) – a velocity of a material point at a particular (instant) moment in time at a particular point of its trajectory.

$$\mathcal{V}_o = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{r}}{\Delta \mathrm{t}} = \frac{dr}{dt}$$

Average acceleration (a_s) – a vector quantity refers to the time change of the velocity vector (its change in size and direction).

$$a_s = \frac{\Delta \mathcal{V}}{\Delta t}$$

$$[a_s] = \frac{[\Delta \mathcal{V}]}{[\Delta t]} = \frac{1m * s^{-1}}{1s} = 1m * s^{-2}$$

Instantaneous acceleration (a_o) – a quantity defined as the first derivative of velocity to time.

$$a_o = \lim_{\Delta t \to 0} \frac{\Delta \mathcal{V}}{\Delta t} = \frac{d\mathcal{V}}{dt}$$

Types of mechanical motion

According to the shape of the trajectory, there are **rectilinear** and **curvilinear** motions. Depending on the time change of the velocity, the motions are **uniform** and **non-uniform (uneven)**.
Uneven motions can be **accelerated** and **slowed**. They are characterized that the value of the instantaneous velocity is increasing (or decreasing) over a certain time to a certain value.

Uniformly accelerated motion

If vectors $\mathcal{V}$ and a have the same direction, then:

$$\mathcal{V}(t) = \mathcal{V}_o + at$$

This motion is called *uniformly accelerated motion*

$$S(t) = \int \mathcal{V}(t)dt = \int (\mathcal{V}_o + at)dt = \mathcal{V}_o t + \frac{at^2}{2} + c \rightarrow (c = S_o)$$

$$S(t) = S_o + \mathcal{V}_O t + \frac{at^2}{2}$$

Uniformly slowed motion

If vectors $\mathcal{V}$ and a have different directions, then:

$$\mathcal{V}(t) = \mathcal{V}_o - at$$

This motion is called *uniformly slowed motion*

$$S(t) = S_o + \mathcal{V}_O - \frac{at^2}{2}$$

Note: we did not use the vector tag for these purposes

Motivation

- **How does kinematic work?**
- **What types of motions do we know?**

Goals

- Draw graphs of position vs. time, velocity vs. time, and acceleration vs. time of a cart rolling on a ramp. Differentiate individual curves in graphs by color and description.
- Determine instantaneous truck velocities in the middle of the path (distance) and in the lowest point (position) of the ramp.
- Determine the truck acceleration.

Scheme of connection

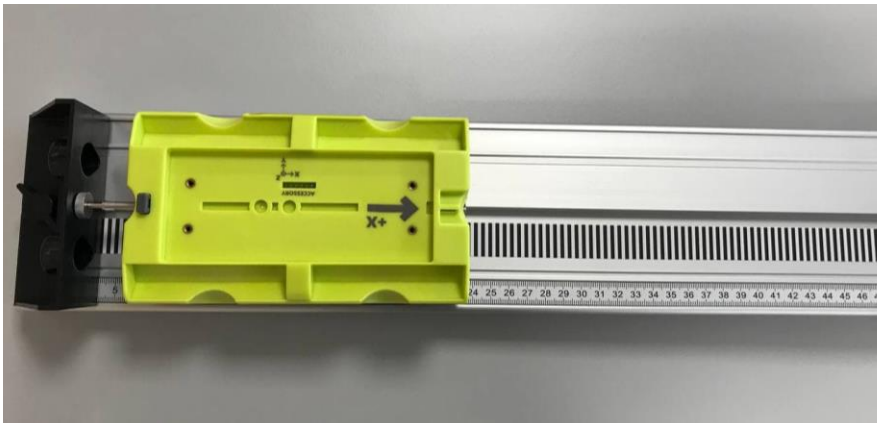
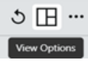


Figure 1

Procedure

Part I.

1. Check the ramp to be inclined at a 3-5 degrees angle.
2. Turn on the cart and place it on the ramp, the plunger must face the end stop (Figure 1).
3. Open the program *Vernier Graphical Analysis* and using *Sensor Data Collection* find your cart and connect it to the system.
4. Click *View Options* , activate *Graph - 3 Graphs, Data Table, Meters* (Figure 2).

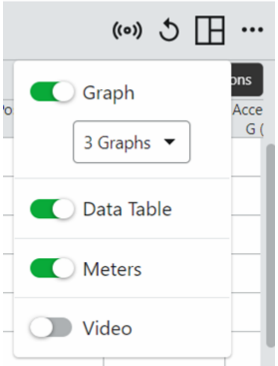


Figure 2

5. Make the cart position zero, clicking on *Zero* (Figure 3).

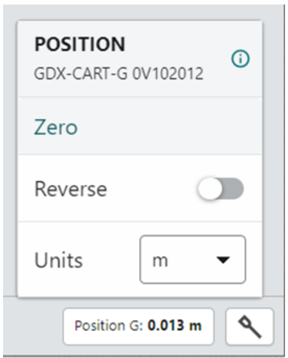


Figure 3

6. Click  to begin data collection. Wait about a second, then briefly push the cart up the incline, letting it roll freely up nearly to the top, and then back down. Catch the cart as it nears the end stop.
7. Examine the position vs. time graph. Repeat Step 4 if your position vs. time graph does not show an area of smoothly changing position. Check with your teacher if you are not sure whether you need to repeat data collection.
8. Answer the Analysis questions for Part I before proceeding to Part II.

Part II.

9. The cart can bounce against the end stop with its plunger. Try starting the cart so it bounces at least three times during data collection.
10. Proceed to the Analysis questions for Part II.

Analysis

Part I.

1. Either print or sketch the three motion graphs. The graphs you have recorded are fairly complex, and it is important to identify different regions of each graph. Move the cursor across any graph to answer the following questions. Record your answers directly on the printed or sketched graphs.
   a. Identify the region when the cart was being pushed by your hand:
      - Examine the velocity vs. time graph and identify this region. Label this on the graph.
      - Examine the acceleration vs. time graph and identify the same region. Label this.

   b. Identify the region where the cart was rolling freely:
      - Label the region on each graph where the cart was rolling freely and moving up the incline.
      - Label the region on each graph where the cart was rolling freely and moving down the incline.

   c. Determine the position, velocity, and acceleration at specific points:
      - On the velocity vs. time graph, decide where the cart had its maximum velocity, just as the cart was released. Mark the spot and record the value on the graph.
      - On the position vs. time graph, locate the highest point of the cart on the incline. Mark the spot and record the value on the graph.
      - What was the velocity of the cart at the top of its motion?
      - What was the acceleration of the cart at the top of its motion?

2. The motion of an object in constant acceleration is modeled by $x = \frac{1}{2} at^2 + v_0t + x_0$, where x is the position, a is the acceleration, t is time a v_0 is the initial velocity. This is a quadratic equation whose graph is a parabola. If the cart moved with constant acceleration, your graph of position vs. time will be parabolic. Fit a quadratic equation to your data.
   a. Click and drag the mouse across the portion of the position vs. time graph that is parabolic, highlighting the free-rolling portion.
   b. Click *Curve Fit* (Figure 4), select Quadratic fit from the list of models (Figure 5), and click *Apply*.
   c. Is the cart's acceleration constant during the free-rolling segment?

3. The graph of velocity vs. time is linear if the acceleration is constant. To fit a line to this data, click and drag the mouse across the free-rolling region of the motion. Click *Linear Fit*, (Figure 6). Compare the slope of the line with the acceleration you found in the previous step.

4. The graph of acceleration vs. time should appear approximately constant during the freely-rolling segment. Click and drag the mouse across the free-rolling portion of the motion and click *Statistics* (Figure 4). Compare the mean acceleration value with the values of a found in steps 2 and 3.

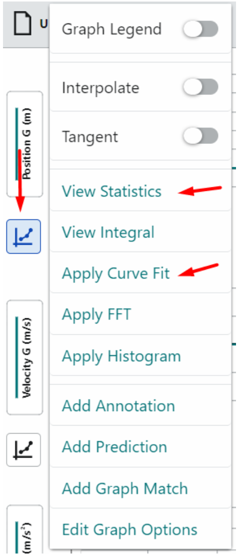


Figure 4

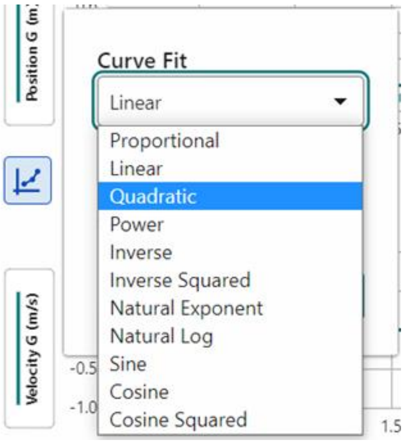


Figure 5

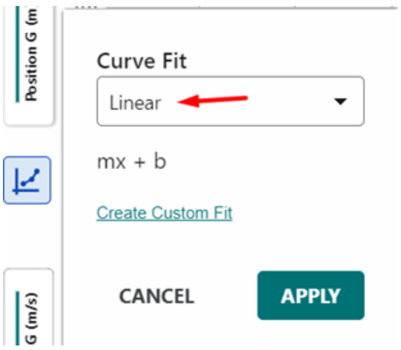


Figure 6

Part II.

5. Determine the cart's acceleration during the free-rolling segments using the velocity graph. Are they the same?

6. Determine the cart's acceleration during the free-rolling segments using the position graph. Are they the same?

Additional questions

What kind of motion did the cart make on the ramp?
Think about it: How does the cart acceleration depend on the inclination of the ramp it is rolling on?

Requisite